FRACTAL DESIGN
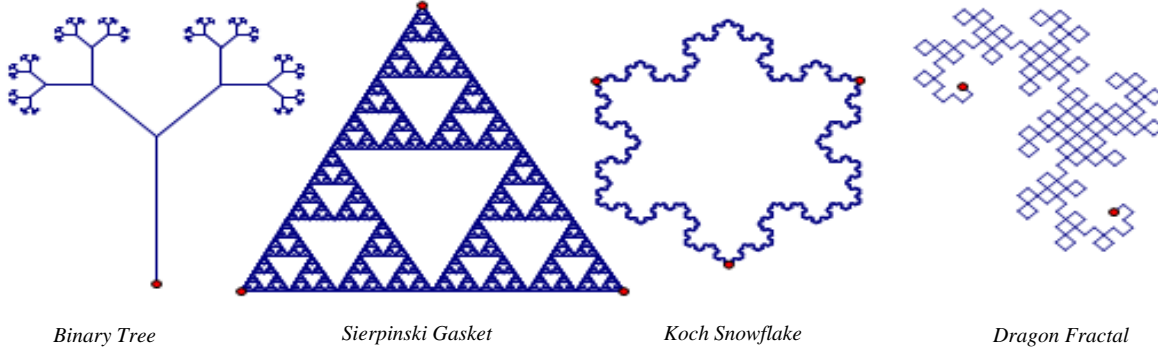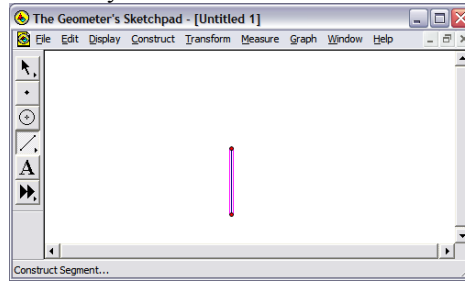

| Binary Tree | Sierpinski Gasket | Koch Snowflake | Dragon Fractal |

Fractals are fun to make and design new ones. Counting the number of new objects in each iteration of a fractal creates a geometric sequence.

BINARY TREE

I.   Create a vertical segment in the middle of your sketch.

II.   Double click on the bottom point until it flashes. This designates that the point has been marked as a center. This can also be done by highlighting the bottom point and selecting **Mark Center** under the **Translate** menu. Next, highlight the top point and select Dilate under the Transform menu.
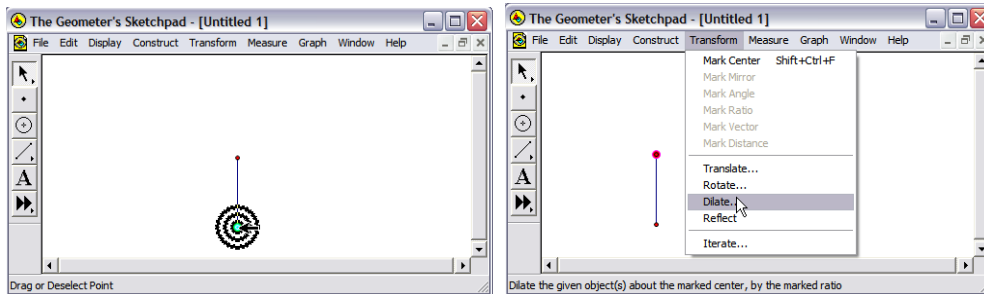

III.   Change the ratio to $\dfrac{3.0}{2.0}$ and click Dilate .
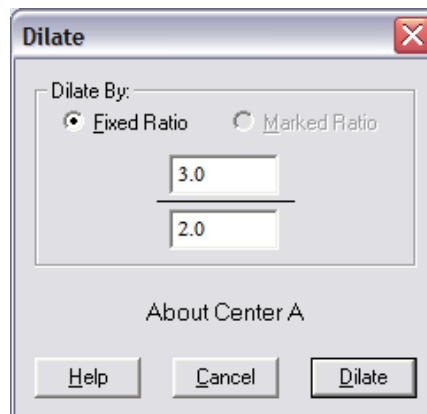
IV.   Mark the top point of the original segment as a Center by doubling clicking on it (you should see it flash). Then, highlight the dilated point and select **Rotate…** under the **Transform** menu.
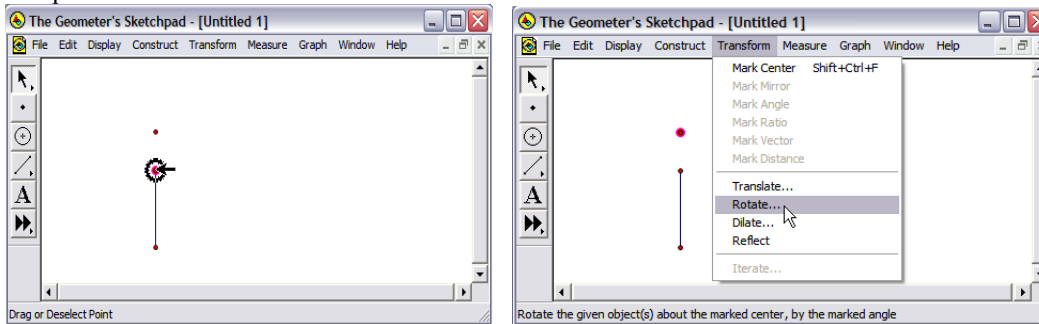

V.   Change the rotation to 45º and press Rotate. Deselect everything by clicking in a blank space. Highlight the original dilated point again. Again, select Rotate under the Transform menu. This time change the rotation to −45º and press Rotate.
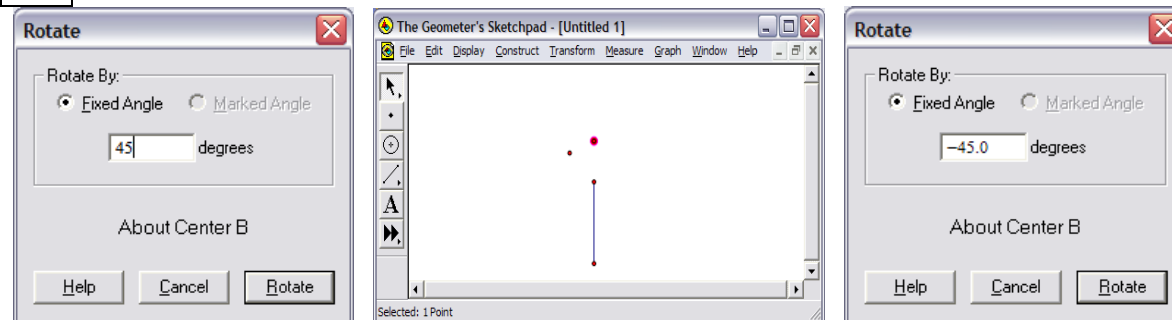
VI.   Again, highlight the dilated point and select **Hide Point** under the **Display** menu.
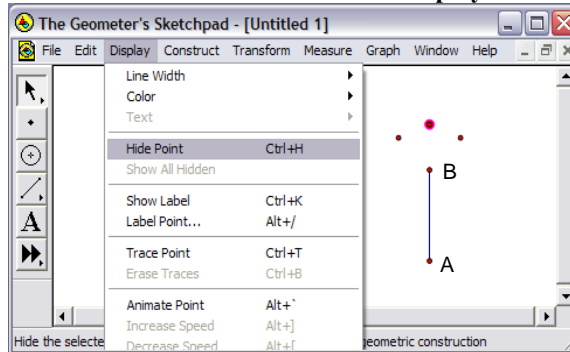
VII.   Next, highlight the bottom point of the original segment (point A) and then the top point of the original line segment (point B). The order in which you highlight the points is important. After highlighting point A and then point B, select **Iterate…** under the **Transform** menu. Next, select the top of the original segment (point B) and the rotated point on the left (point C). To finish the iteration, click on the Structure button and select **Add New Map**. Then, click on the top point of the original segment again (point B) and the rotated point on the right (point D). The number of iterations can be increased or decreased by clicking on the + button or the − button. Finally, click on the **Iterate** the button.
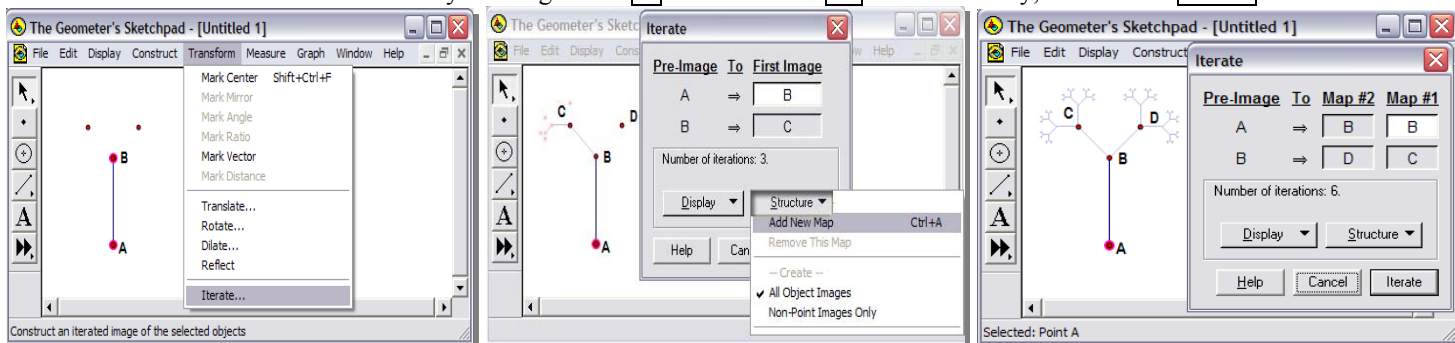

VIII.   Possible Extensions: List the number of new segments in each iteration. What type of sequence is this called?
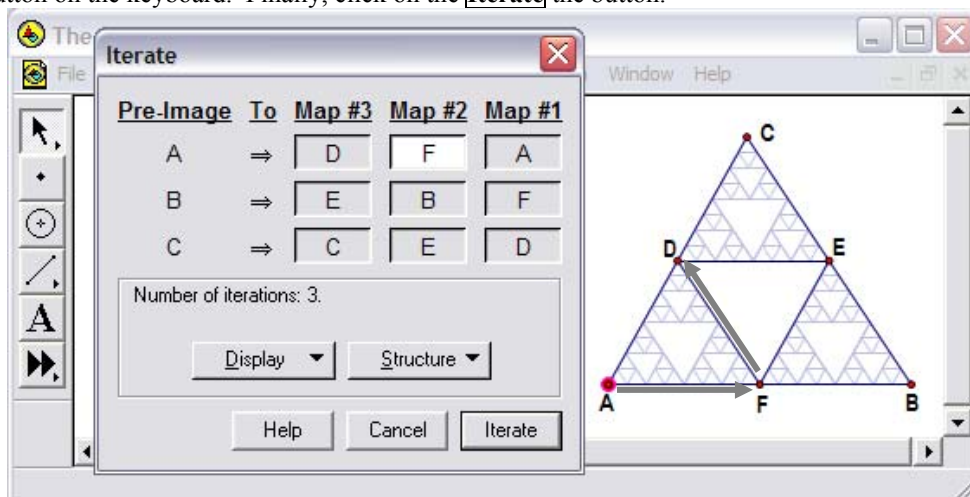
A Quick Sierpinski Gasket using Segments

I. Using the segment tool create a triangle. Then, using the selection tool highlight all three sides of the triangle and select Midpoints from the Construct menu.
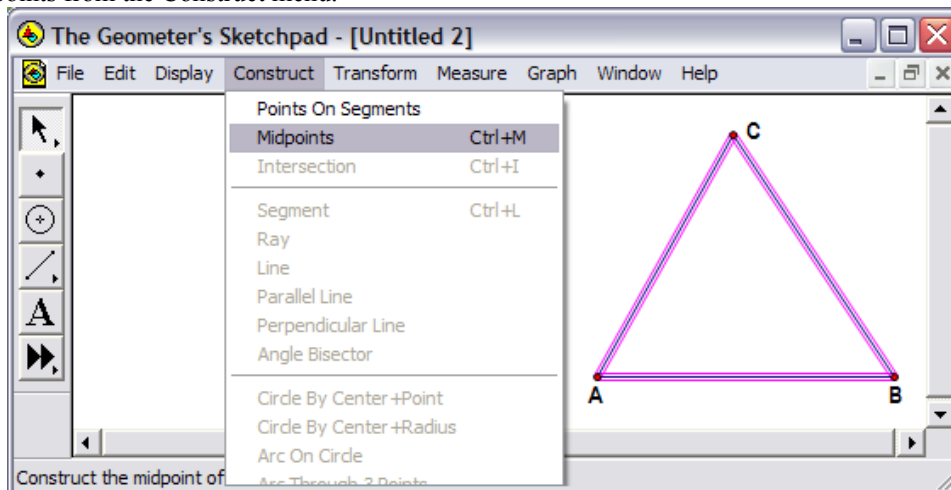
II. Highlight all three new midpoints and select Segment under the Construct menu.
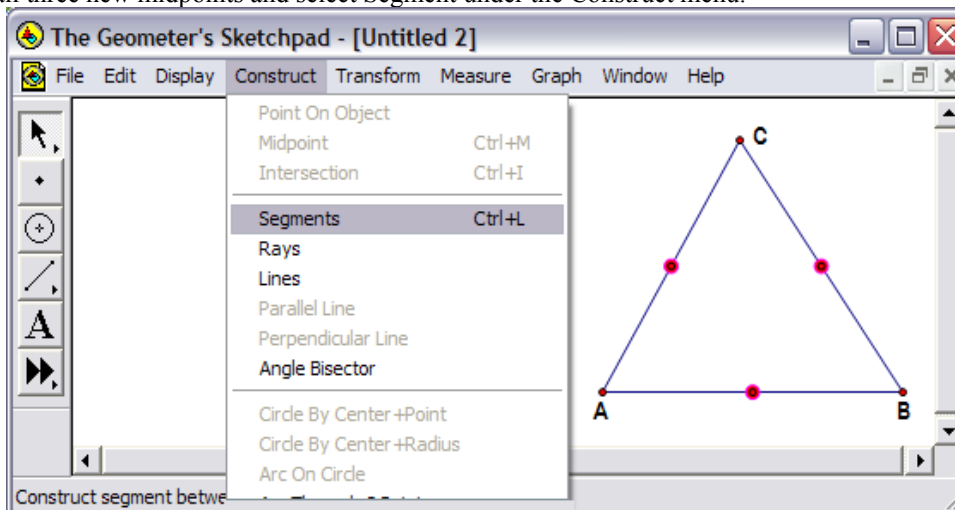
III. Highlight the original points A, B, and C in that order and select **Iterate** under the **Transform** menu. Since the points were selected in that order each iteration needs to be selected in the same order. In other words, since we selected A, B, and C, we need to select points A, F, and D so that the points are mapped properly. Then, under **Structure** select **Add New Map** and select points F, B, and E. Again, under **Structure** select **Add New Map** and select points D, E, and C. The number of iterations can be increased or decreased by clicking on the [+] button or the [−] button on the keyboard. Finally, click on the **Iterate** the button.

IV. Make a list of the number of new triangles created after each iteration. What type of sequence is generated?

## A Quick Sierpinski Gasket using Triangle Interiors

o Start a New Sketch in Geometer's Sketchpad

o Create a triangle using the segment tool as shown in the diagram. Deselect everything by clicking in a blank area. Next, highlight all three segments that create the triangle. Under the **Construct** menu select **Midpoints**.

o Deselect everything by clicking in a blank area. Highlight all of the segments again and select **Hide Segments** under the **Display** menu.

o Highlight the vertices of the original triangle and select **Triangle Interior** under the **Construct** menu. Change the color of the interior to your preference under the **Display** menu.

o **(The order of the highlighting is critical in this step)**. Deselect everything by clicking in a blank area. Highlight the points A, B, and C in that order. Next, select **Iterate…** under the **Transform** menu. This will bring up an window that will request a mapping of the first image. *(see the diagrams below for help)*

o **(The order of the highlighting is critical in this step)**. In order to repeat the right mapping the new smaller triangle's vertices must be selected in the exact same order. First, map the smaller triangle in the lower left-hand corner. Click on A,D, and E in that order. *(see the diagrams below for help)*

o Next, add a new mapping by clicking on the Structure ▼ button and selecting **Add New Map**. This will show a new column for a new mapping of a Map#2 . This time try mapping the top triangle by clicking on the points E,F, and C in that order. *(see the diagrams below for help)*

o Finally, create yet another mapping for the bottom right triangle and selecting points D,B, and F in that order.
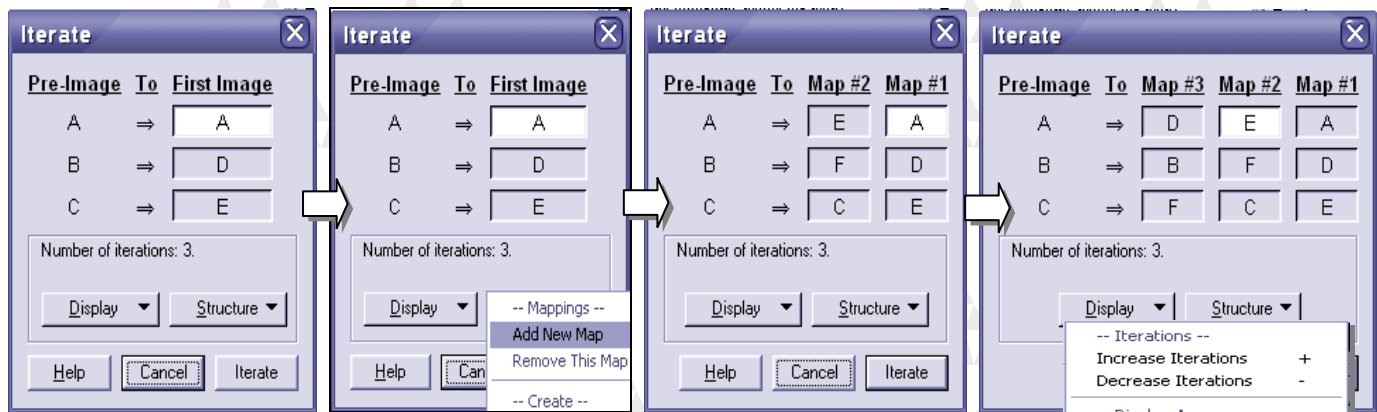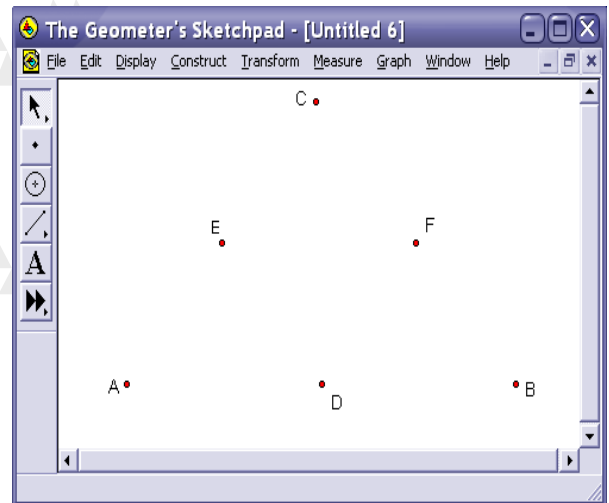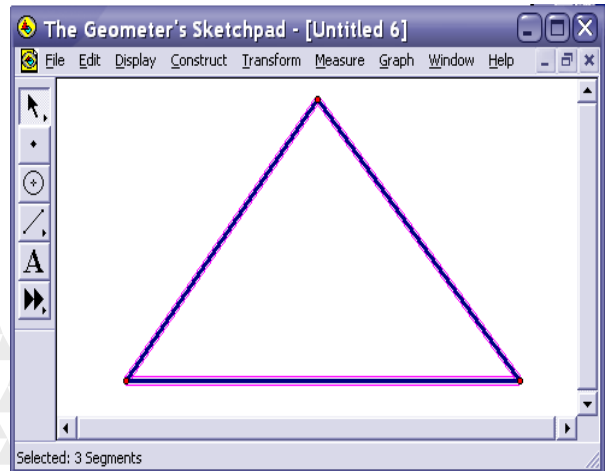
o Next, under the Display ▼ button you can increase the number of iterations and more importantly the **FINAL ITERATION ONLY** needs to be checked. Click on Iterate .

o Finally, highlight the original triangle interior and select **Hide Triangle** under the **Display** menu.

A *Koch Snowflake*

I. Start by creating a horizontal line segment. Double click on the left point (point A) which will mark it as a center.
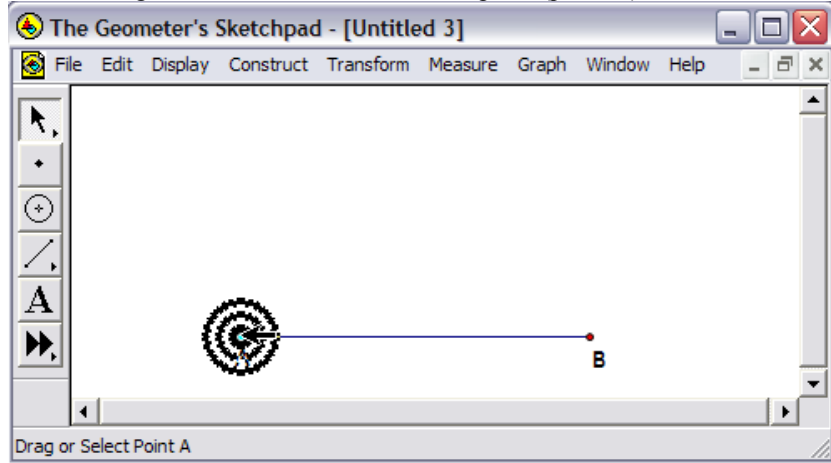

II. Next, highlight the right endpoint (point B) and select **Dilate** under the **Transform** menu. Change the ratio to 1/3 and **Dilate**. After the new image of point is created, deselect everything by clicking in a blank space and highlight the point B again. Again, select **Dilate** under the **Transform** menu. Change the ratio to 2/3 and **Dilate** .
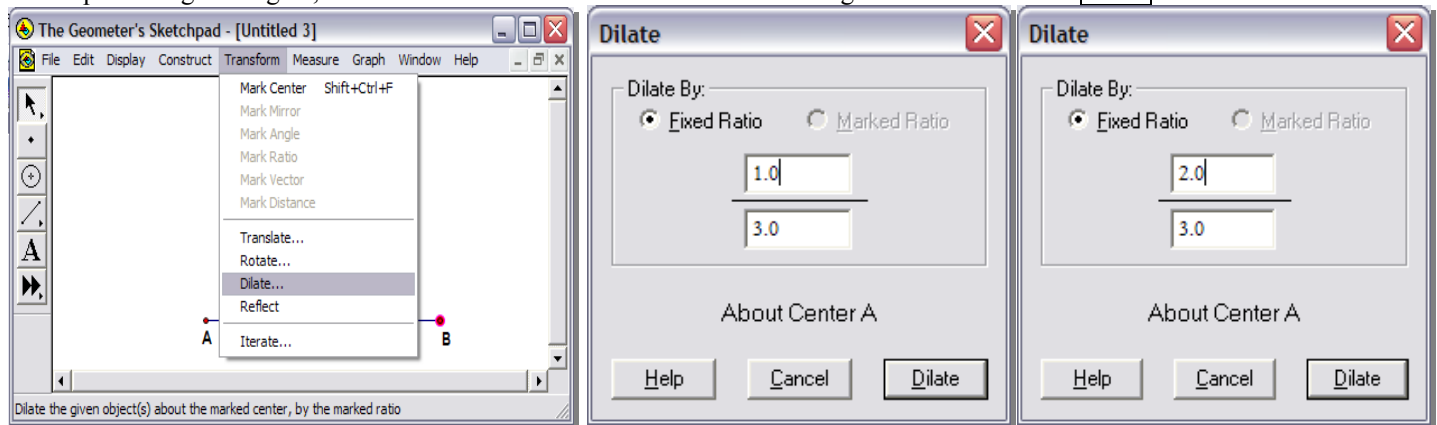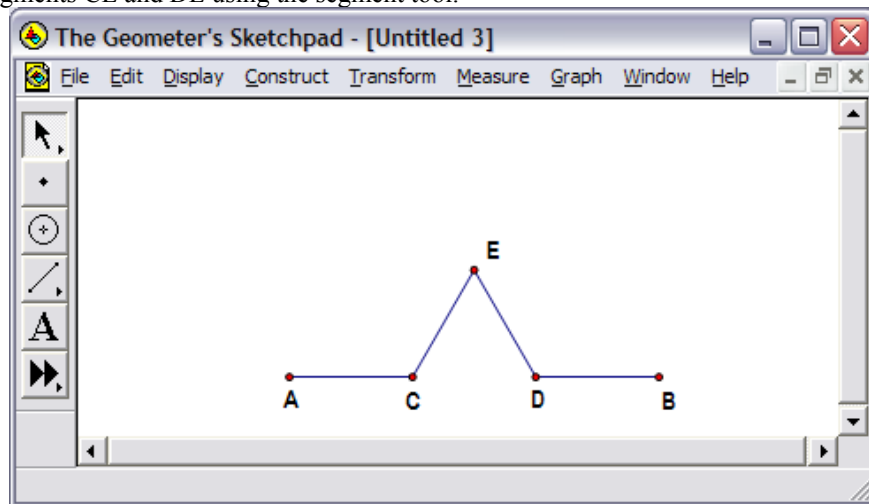

III. Highlight the segment AB and select **Hide Segment** under the **Display** Menu. Using the segment tool create the segment AC and DB shown below.
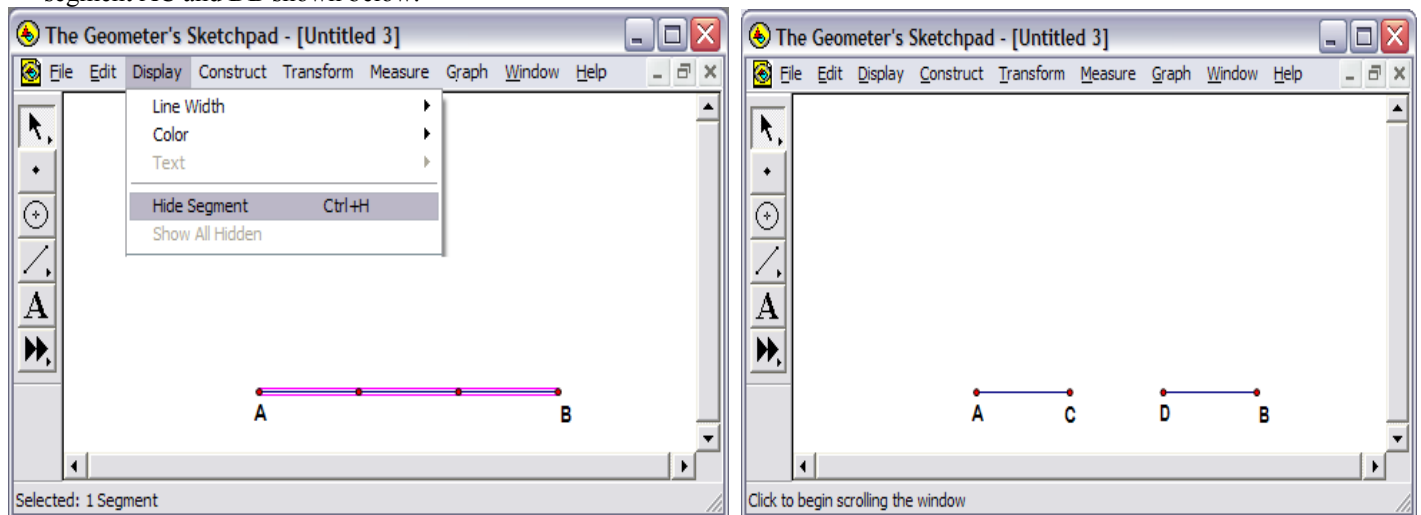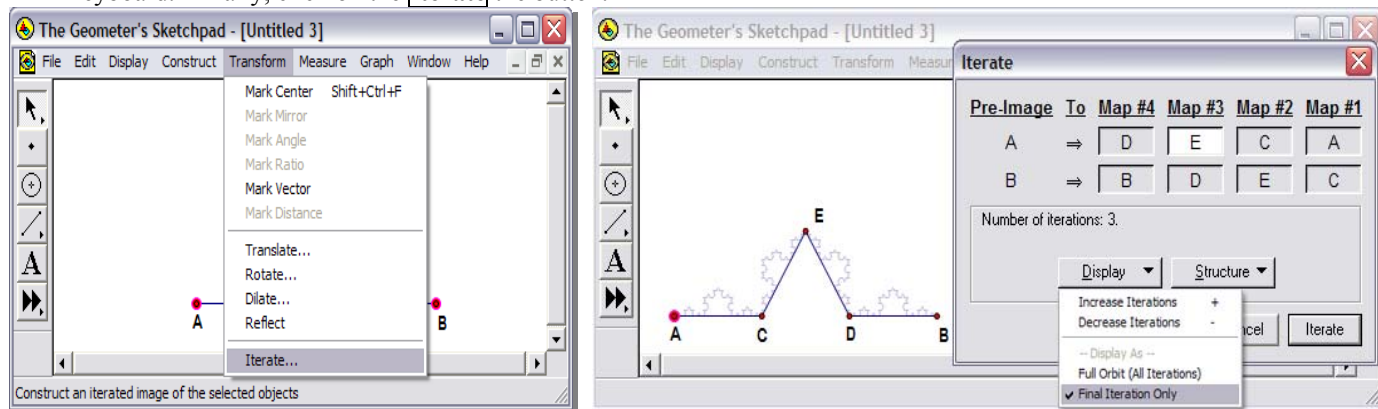
IV. Double click on point C to mark it as a center. Highlight point D and select Rotate under the Transform menu. Change the angle to 60º and press Rotate.
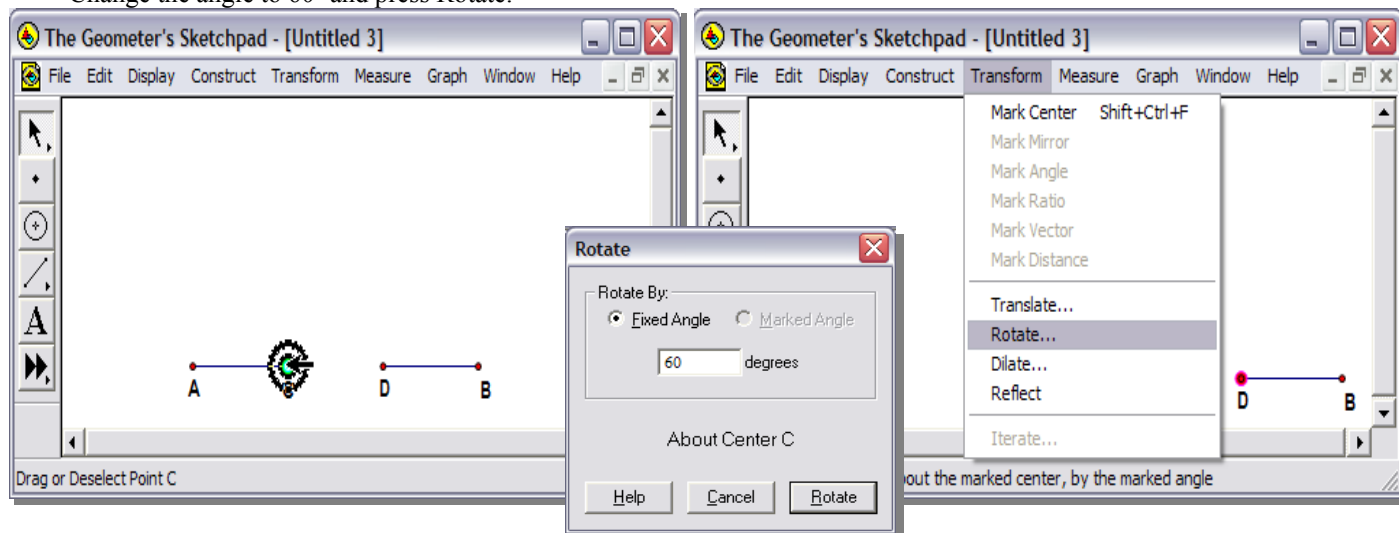

V. Create the segments CE and DE using the segment tool.

VI. Highlight points A and B in that order. Select Iterate from the Transform menu. Start with the mapping A and C in that order. Then, under the **Structure** button choose **Add New Map** and select points C and E. Repeat the process of adding new maps for E & D and D & B. Under the **Display** button select Final Iteration Only. The number of iterations can be increased or decreased by clicking on the $+$ button or the $-$ button on the keyboard. Finally, click on the **Iterate** the button.


VII. Highlight the original segments AC, CE, ED, DB and the points C,E,D. Select Hide Objects under the Display menu.

VIII. The Geometer's Sketchpad version 4 is not able to rotate iterations. Instead, we will need to create a custom tool of the first part of the snowflake. Highlight the entire sketch by dragging a box around the first part of the snowflake (shown below). Click on the custom tool button, , and select **Create New Tool….** Change the Tool Name to something like "Snowflake" and press OK .
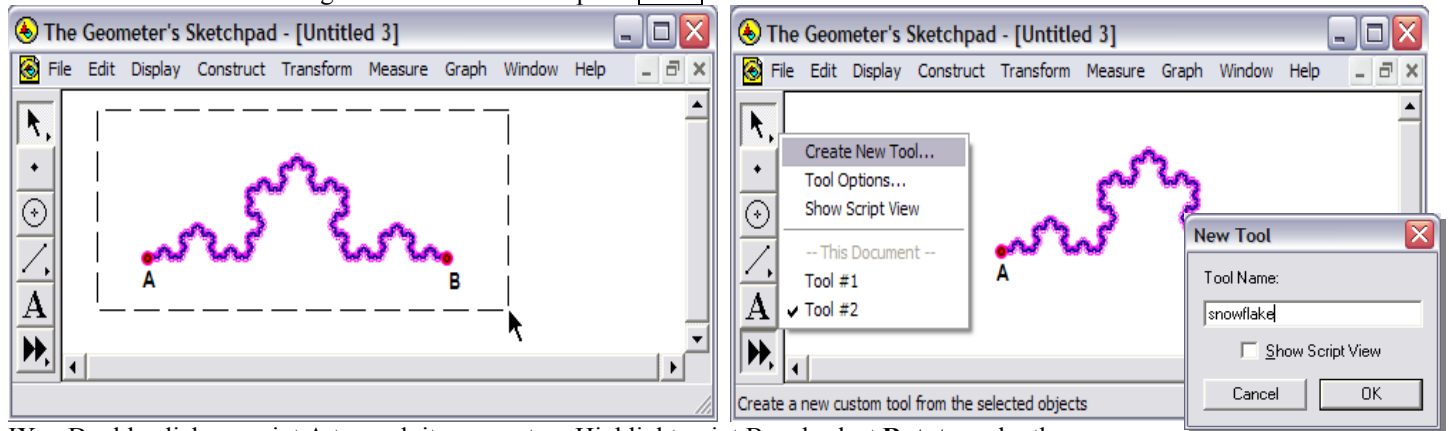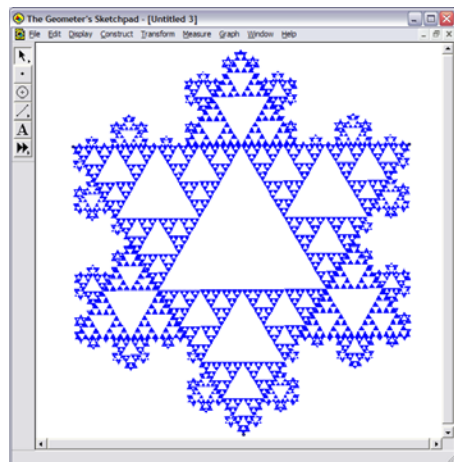

IX. Double click on point A to mark it as a center. Highlight point B and select **Rotate** under the **Transform** menu. Change the angle to **−60º** and press Rotate . Label the new point F. Go to the custom tool button and select your new tool "Snowflake". With your "Snowflake" tool click on point B and then the new point F. Then, click on the point F followed by point A to finish creating the snowflake.
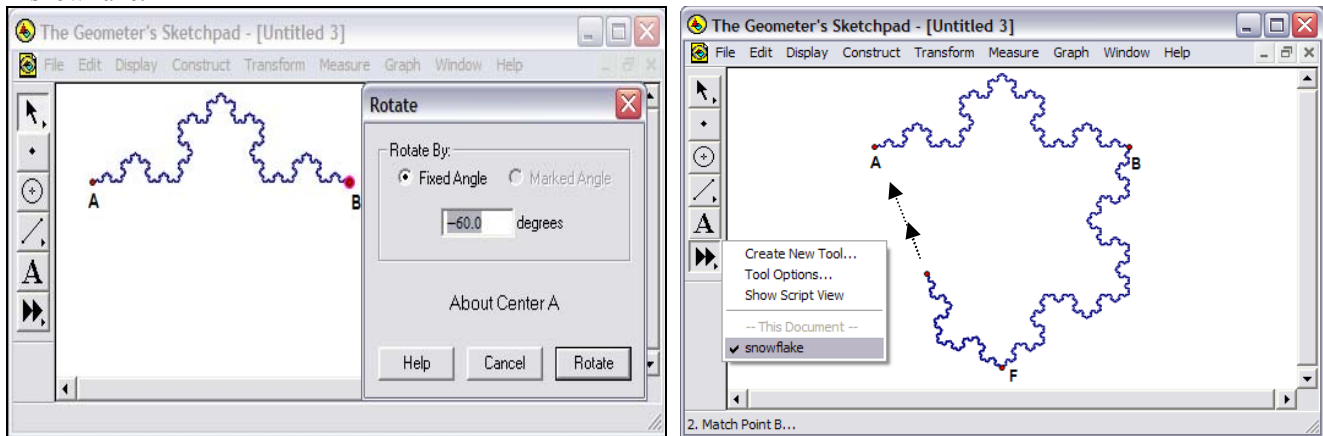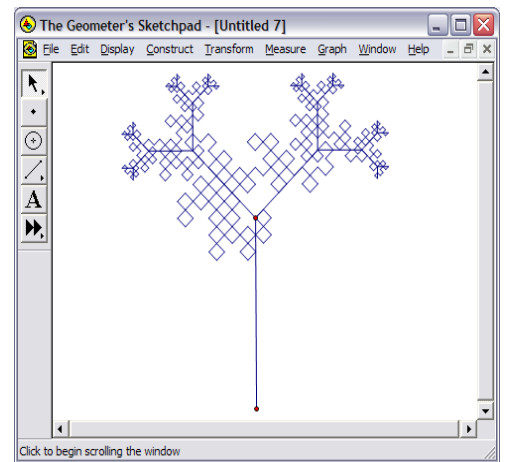

X. In a very similar manner, it is possible to create Koch snowflake using triangle interiors instead of segments (see example below). It is also possible to mix some fractals together (see *Sierpinski-Kosch Snow Gasket*) . With each iteration how many new objects are created by the Koch snow flake?
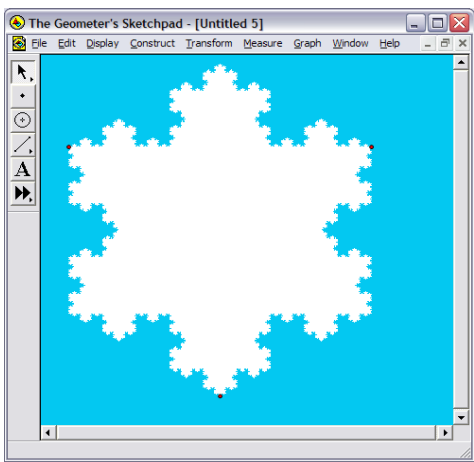
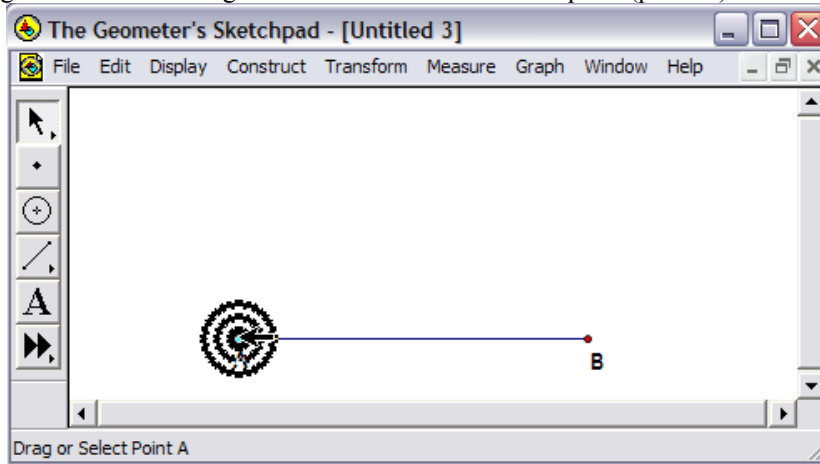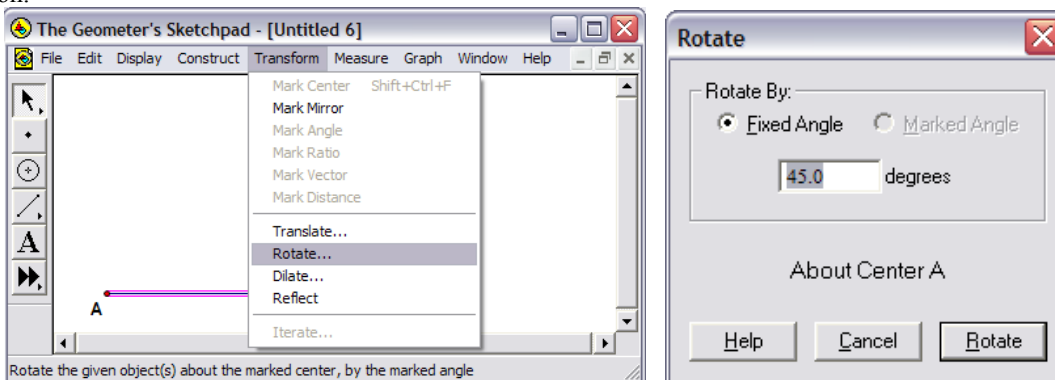**Koch Snowflake Polygon**   **Sierpinski-Kosch Snow Gasket**   **Binary Dragon Tree**

The Dragon Fractal

I.   Start by creating a horizontal line segment. Double click on the left point (point A) which will mark it as a center.


II.   Highlight segment AB. Select **Rotate** under the **Transform** menu. Change the angle to **45º** and press the **Rotate** button.

III. This time double click point B. Again, Highlight segment AB. Select **Rotate** under the **Transform** menu. Change the angle to **−45º** and press the **Rotate** button.
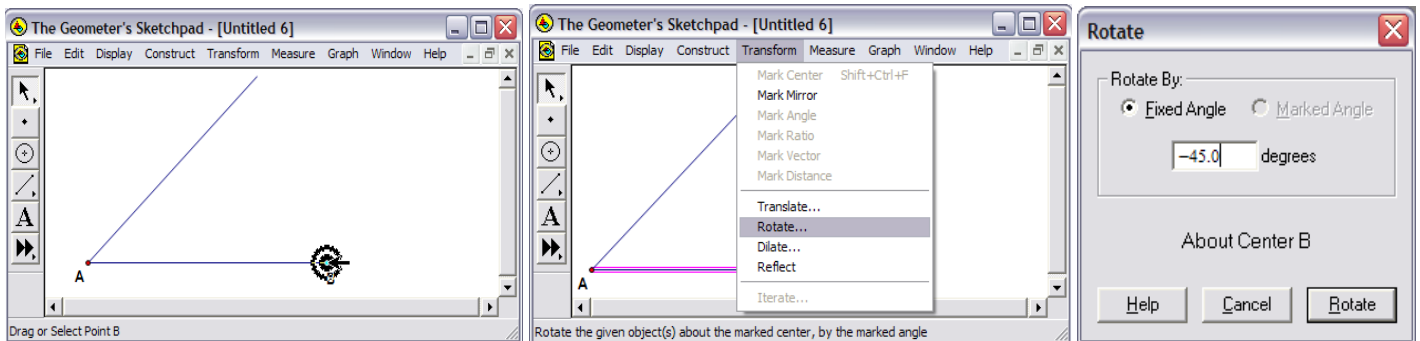

IV.   Using the selection tool click precisely on the intersection of the two rotated segments. This should create a point of intersection.
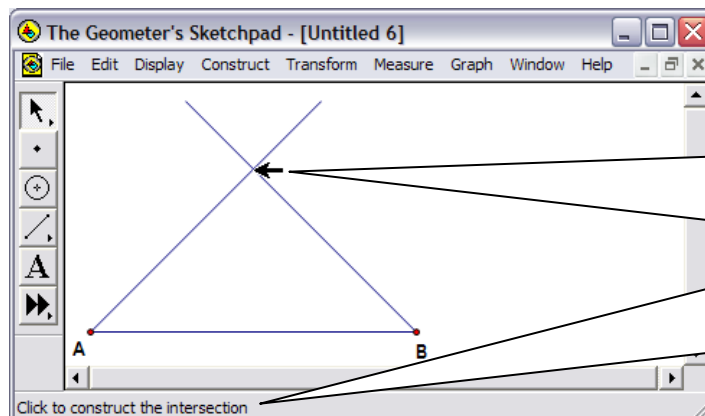

Click on the precise place of intersection of the two rotated segments to create a point of intersection. Notice the message bar informs you of what will happen if you click.

V. Click in a blank space to deselect everything. Highlight all of the segments on the sketch and select **Hide Segments** under the **Display** menu. Then, create segments AC and BC with the segment tool as shown below.
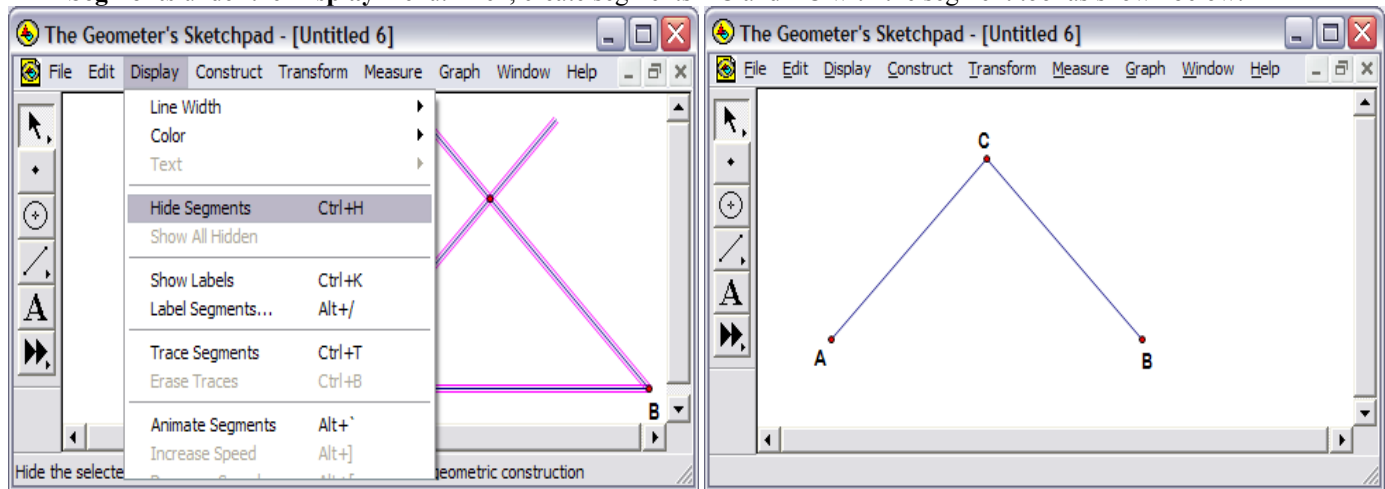
VI. Highlight points A and B in that order (order is important when creating iterative mappings) and select **Iterate…** under the **Transform** menu.
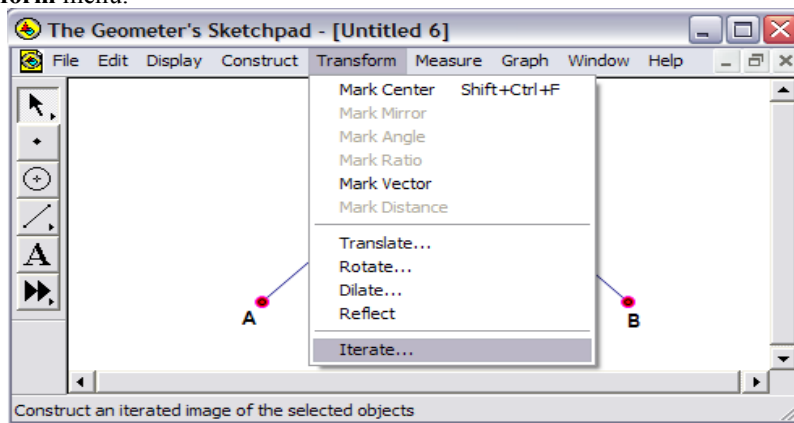

VII. First position the Iterate window so that you can still see the sketch. Next create the first mapping by clicking on point A and then point C. Click on the Structure button and select Add New Map. For the last mapping, click on Point B followed by Point C (order is critical). Under the Display button select "**Final Iteration Only**". The number of iterations can be increased or decreased by clicking on the $+$ button or the $-$ button on the keyboard. Finally, click on the Iterate the button.
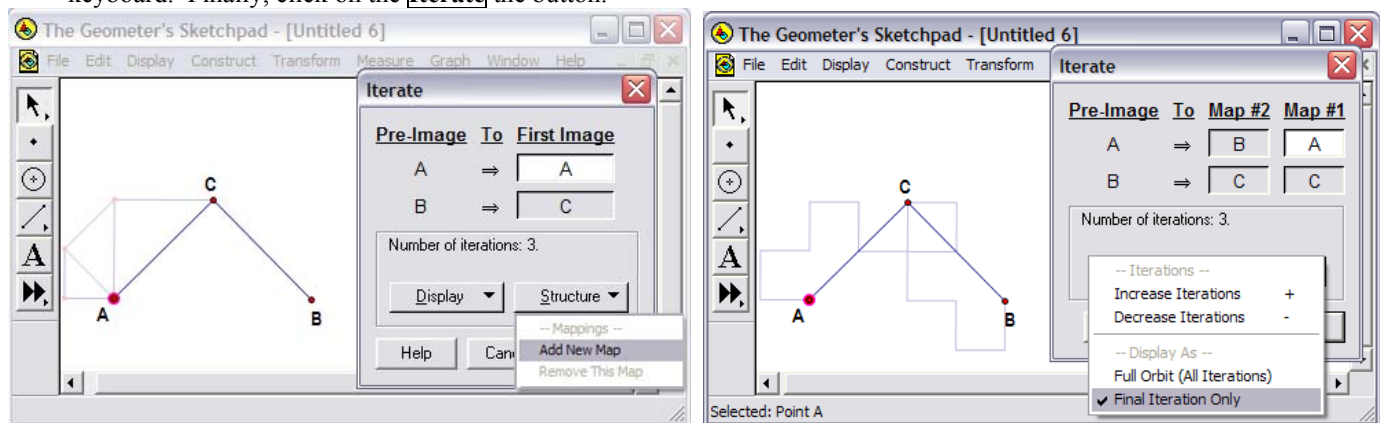
VIII. Highlight the original segments AC and BC. Select **Hide Segments** under the **Display** menu.

Try Experimenting with either altering some of these fractals or even start from scratch and make some new fractals. Don't forget to consider the investigating the geometric sequence generated by the number of objects in each new iteration.
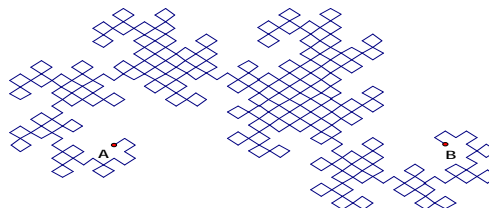
The golden ratio, phi (φ), appears all throughout nature. This ratio appears in pine cones (count the number of rows going in one direction and compare it to the number going in the other direction), in the way a lightning bolt branches, the way in which a river branches, time and time again in skeletal structures, and in all crystal structures (You may want to try experimenting with copying and pasting pictures into sketchpad and verifying the ratio).

This ratio is mathematically defined by: $\dfrac{a}{1} = \dfrac{1}{1+a}$ where a is the golden ratio and suggests

that $a \approx 1.61803$. This ratio can also be defined by $\dfrac{a}{b} = \dfrac{b}{b+a}$ which suggests that this

can be created quickly with sketchpad.

- **Start by constructing a square ABCD.**
    - Open a new sketch
    - Create a horizontal line segment using the segment tool
    - Double click on the left endpoint (point A). The point should flash as seen at the right. This will mark the endpoint as a center.
    - Highlight the right endpoint (point B). Select **Rotate…** under the **Transform** menu. Change the angle to 90 degrees and press the Rotate button.
    - Next, double click on the right endpoint (point B). The point should flash and this will mark the right endpoint as the new center.
    - Highlight the left endpoint (point A). Select **Rotate…** under the **Display** menu. Change the angle to –90 degrees and press the Rotate button.
    - Highlight all four points. Select **Segments** under the **Construct** menu (pressing CTRL and L at the same time is a keyboard shortcut for this command).
- Next, create the midpoint of AB by highlighting the segment AB and selecting Midpoint. Label the midpoint Point E.
- Create a circle centered at point E and passing through point D.
- Change to the line tool and create a line (not a segment) through point A and Point B. Find the intersection point of line AB and the circle. Label the intersection point F.
- Highlight point F and line AB. Select **Perpendicular Line** under the **Construct** Menu.
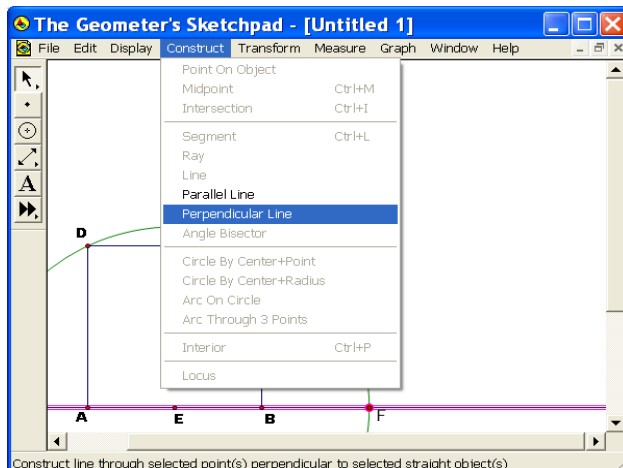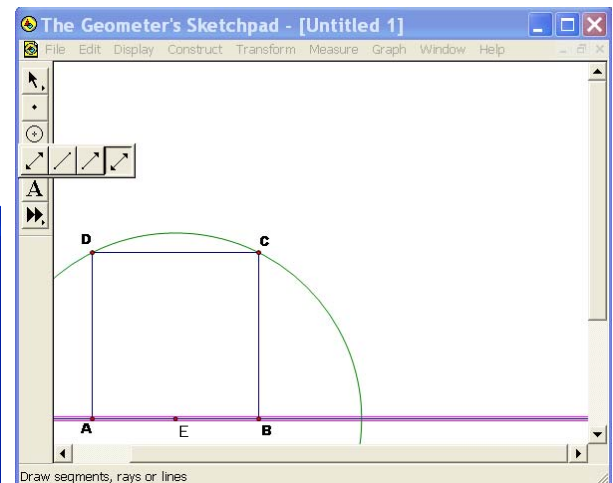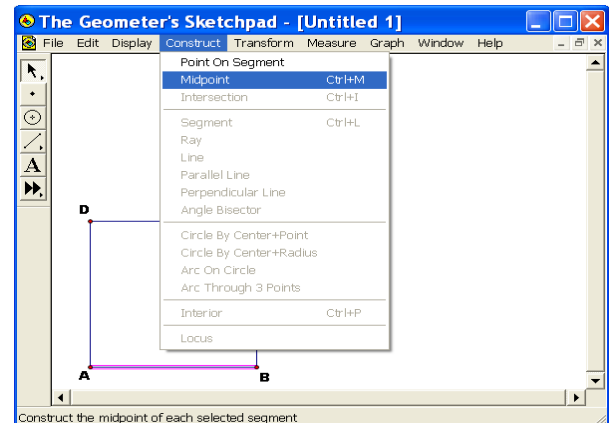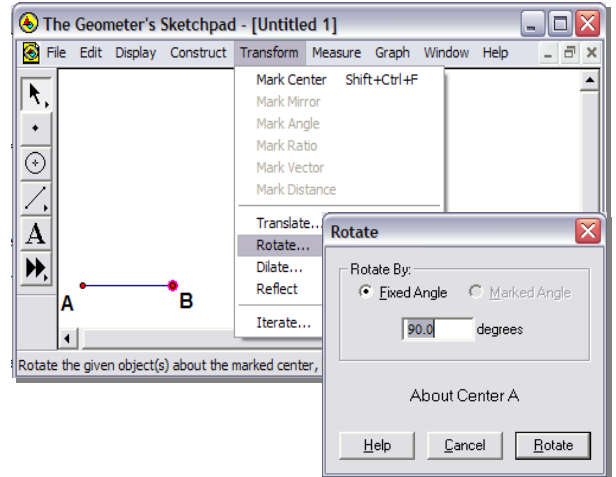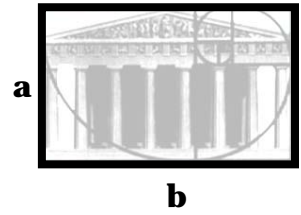
- Change to the line tool and create a line (not a segment) through point D and Point C. Find the intersection point of line DC and the newly created perpendicular line. Label the intersection point G.

- Hide all of the segments, circles and lines except segment BC . Re-draw the rectangle by connecting the vertices A,D,G, and F with segments.

- Create a circle centered at point C and passing through point B.

- Highlight the circle and points D and then B in that order. Select **Arc On Circle** under the **Construct** menu.

- Highlight just the circle and select **Hide Circle** under the **Display** menu.

- Create a circle centered at point F and passing through Point B. Using the selection tool click on the intersection of the new circle and segment FG. Label this intersection point H.

- Hide the Circle

- Highlight points A and B in that order. Select **Iterate…** under the **Transform** menu.

- Map point A to point F and point B to point H. Next select **Non-Point Images Only** under the **Structure** button.

- Finally, click **Iterate**. Click the "+" or "-" keys on the keyboard to increase or decrease the number of iterations of the spiral.